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Selecting weights to evaluate the effectiveness of basketball players with DEA

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Abstract

In this paper we show how DEA may be used to identify component profiles as well as overall indices of performance in the context of an application to assessments of basketball players. For this purpose we go beyond the usual uses of DEA to provide only overall indexes of performance and extend this by focusing on the component values, including the multiplier values for the efficiently rated players. For this purpose a new procedure is used that guarantees a full profile of non-zero components of the “multipliers (or “weights”) including the “virtual values,” as they are referred to in the DEA literature. We demonstrate their use by providing profiles that can be used to identify relative strengths and weaknesses in individual players. We also utilize the flexibility of DEA in the sense that we may specify conditions on the weights (or bounds) that reflect the views of coaches, trainers and other experts on the basketball team for which the evaluations were conducted. Finally we show how these combinations can be extended by taking account of team as well as individual considerations for similar uses in other contexts.

Keywords: Indexes of Performance, Data Envelopment Analysis (DEA), Efficiency, Effectiveness, Weights, Basketball.

1. Introduction

In this paper we address the problem of developing indexes of performance that result from aggregation of several indicators by using DEA. These indexes generally have the form of a weighted sum of the variables. It sometimes happens that experts, by virtue of their knowledge of the problems, are able to prescribe a set of weights, which (unlike in DEA) are common to all DMUs. However, an issue often arises in practice because the weights that are
to be assigned to different inputs and outputs are unknown for such a priori specification. In addition, the weights, if used, may be unsatisfactory in terms of their further effects.

As is well known, DEA yields efficiency scores in the form of a weighted sum of either several inputs or several outputs without any need for a priori information on the relative values of these variables. In addition, DEA permits incorporating “bounds” on the weights and thus relaxes the need for precise knowledge of values for the weights to be used. This all suggests that DEA might provide a useful methodology to improve presently used indexes of performance in very flexible ways.

In the development of DEA, the possibility of providing efficiency indexes without any need to have information available about costs and prices or other preassigned weights was emphasized and pointed up as one of the attractive features of this new methodology. In place of preassigned weights, DEA determines their values in a manner that maximizes the efficiency score of the entity being evaluated. This allows great flexibility in the choice of the weights. As a consequence, the resulting weights may sometimes be unreasonable, and they may not be consistent with accepted views. See, e.g., Allen et al. (1997) and Thanassoulis (2001) for purposes, motivations and uses of weights restrictions or bounds on admissible values.

Here we will use an approach described in Cooper, Ruiz and Sirvent (2005) which provides another (new) way to choose weights for use in DEA. This approach involves a two-step procedure that effects a choice of weights that are associated with alternative optimal solutions of the extreme efficient DMUs provided by dual, “multiplier,” formulations of the DEA model. Among other things, for efficient units this procedure guarantees a set of strictly positive weights for all inputs and outputs and thus eliminates the need for dealing with zero values for these weights.

In this paper we provide an example of the use of this approach in the form of an application to the assessment of basketball players. This kind of evaluation is already being done in the Spanish Basketball League (called the ACB league) which uses an index of player assessments that has the form of a unweighted sum of the classical indicators in basketball statistics (points, rebounds, assists, fouls, etc.). The idea here is to examine DEA as a possible alternative to specify weights that can go with these indicators. To be more specific, in this application we exploit the ability of DEA both to (1) set conditions on the relative value of the indicators based on expert opinion by means of restrictions on the
weights and (2) allow the weights to vary across players in order to reflect their different characteristics. This paper is thus in the line of papers like those by Melyn and Moesen (1991), with the so-called “benefit of the doubt” weighting, Takamura and Tone (2003) and Lauer et al. (2004). (The latter two papers also use AHP (the Analytic Hierarchy Processes), to specify bounds on the weights).

One point needs to be made in that we use weight restrictions, such as those provided by the “Assurance Region” (AR) approaches of Thompson et al. (1986), which allow us to specify player profiles by means of upper and/or lower limits for the ratio of the weights that are to be assigned to the performance attributes of the players. Thus, the players are assessed in a manner that is consistent with the views of the coach and his staff and take also account of the different positions the players occupy.

As stated in the above discussion, we also use DEA to analyze the resulting indexes in terms of relative evaluations in a manner that can provide additional useful information in the context of applications to basketball. In particular, we here use DEA to provide insight into (a) how the efficient players achieved their efficiency and (b) which aspects of their game the deficient players can improve.

We also depart from the more customary “efficiency” analyses and focus instead on “effectiveness” in the sense that in our analysis there is no reference to resources consumed, as in the efficiency evaluations. Thus, we follow Prieto and Zofio (2001) and confine attention to player outputs such as points scored and/or percentage of free-throw successes and leave out of consideration such things as player salaries, etc. Similarly we do not include benefit measures such as revenues earned, or like considerations. We confine attention to player qualities that take account of their positions on a team without respect to audience appeal, etc.

DEA, as well as parametric techniques for efficiency analysis, have been used previously in the world of sports. For example, Sueyoshi et al. (1999) and Fried et al. (2004) apply DEA models to players of baseball and golf. Sexton and Lewis (2003) do the same with Major League Baseball teams in the U.S. Zak et al. (1979) analyze the efficiency of basketball teams of the NBA in the U.S. with Cobb-Douglas production functions. Relative efficiency in sports has also been measured at country levels, as in Lozano et al. (2002) where DEA is used to measure the performances of the nations participating at the Summer Olympics. Applications can also be found where efficiency is analyzed from the perspective
of managers of sport teams. For instance, Scott et al. (1985) utilize Cobb-Douglas frontiers to examine the salary and marginal revenue products in professional basketball and Fizel and D’Ittri (1999) carry out an analysis of managerial efficiency with DEA models in order to investigate the impact of firing and hiring managers on organizational performance.

The paper is organized as follows: In section 2 we include some of the theory underlying the use of DEA to develop efficiency indexes in the context of the problems we address. We focus on the specification of weights while paying special attention to the choice of weights between alternative optimum solutions in DEA formulations. For this purpose we briefly describe the procedure proposed in Cooper, Ruiz and Sirvent (2005) for the selection of weights. In section 3 we apply these procedures to the assessment of basketball players in the context of the Spanish Premier League. Section 4 concludes.

2. Theoretical aspects

As already stated, we propose a use of DEA to develop indexes of performance. In general, if we have a set of \( n \) decision making units (DMUs) that use \( m \) inputs to produce \( s \) outputs, we can carry out an analysis of their relative efficiency by either of the following pair of dual problems,

\[
\begin{align*}
\max \ & \phi_v + \epsilon \left( \sum_{i=1}^{m} s_{io}^- + \sum_{r=1}^{s} s_{ro}^+ \right) \\
\text{subject to} \ & \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io}^- - s_{io}^-, \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{ij} = \phi_v y_{io} + s_{ro}^+, \quad r = 1, \ldots, s \\
& \lambda_j \geq 0, \quad j = 1, \ldots, n \\
& s_{io}^- \geq 0, \quad i = 1, \ldots, m \\
& s_{ro}^+ \geq 0, \quad r = 1, \ldots, s,
\end{align*}
\]

(1)

where \( \epsilon > 0 \) is a non-Archimedean element, smaller than any positive real number, and
\[
\begin{align*}
\min \omega_o &= \sum_{i=1}^{m} v_i x_{io} \\
\text{subject to} & \\
\sum_{r=1}^{s} \mu_r y_{ro} &= 1 \\
- \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} \mu_r y_{ijn} &\geq 0 & j = 1, \ldots, n \\
v_i &\geq \epsilon & i = 1, \ldots, m \\
\mu_r &\geq \epsilon & r = 1, \ldots, s 
\end{align*}
\]

where the \( x_{ij} \) and \( y_{ijn} \) represent the input and output amounts recorded for \( DMU_j \), \( j = 1, \ldots, n \), and \( x_{io} \) and \( y_{ro} \) represent these input and output amounts for the \( DMU_o \) being evaluated.

Note that these are, respectively, the primal “envelopment” formulation and the dual “multiplier” formulation corresponding to the so-called CCR (Charnes et al., 1978) output-oriented DEA model.

In this paper the above models are modified to construct an index of performance and identify its components for use in analyzing the components of this index. The thus weighted variables are applied only to the outputs. No inputs are used. Formulations (1) and (2) are thereby simplified, because we include only one constraint in which the input of each DMU is represented as a scalar with unit value. Thus, as already discussed, our measures are directed to measures of “effectiveness” rather than “efficiency”.

We are particularly interested in formulation (2), since the chief purpose of this paper is the use of DEA models to select component weights. For this purpose, Cooper, Ruiz and Sirvent (2005) point up the need for addressing the problem of selecting weights between alternative optimal solutions of the dual multiplier formulation of the DEA model as part of a procedure for choosing a set of weights. It is well-known that the optimal solutions of (1) for extreme efficient units are usually highly degenerate, which lead (2) to have alternate optima. This is a problem when interpreting the relative value of the variables in the efficiency assessment, because the optimal weights may result from selecting only one of these alternatives, as is done by most of the DEA computer codes, and the weights may differ from one optimum to another and thus lead to portrayals of performance that depend only on the
Moreover, it often occurs that only a few weights are non-zero in an optimal solution. The DMU under analysis is then being assessed only on a small subset of the inputs and outputs.

In Cooper, Ruiz and Sirvent (2005) a two-step procedure is proposed for the selection of weights that is based on two general criteria of selection and is implemented by means of two mixed integer linear programming (MILP) problems. We briefly describe this procedure in the following. First we focus on only the “extreme efficient DMUs.” I.e., the DMUs that are in the class E designated in Charnes et al. (1991). For members of this class, E, of extreme efficient DMUs, the following MILP problem is solved in the first step

\[
\min b_o = \sum_{j \in E} b_j \\
\text{subject to} \\
\sum_{r=1}^{s} \mu_r y_{rO} = 1 \\
- \sum_{i=1}^{m} v_i x_i + \sum_{r=1}^{s} \mu_r y_{rO} + t_j = 0 \quad j \in E \\
\sum_{i=1}^{m} v_i x_{iO} = 1 \\
t_j - Mb_j \leq 0 \quad j \in E \\
b_j \in \{0,1\} \quad j \in E \\
v_i \geq 0 \quad i = 1,\ldots,m \\
\mu_r \geq 0 \quad r = 1,\ldots,s \\
t_j \geq 0 \quad j \in E.
\]

Here both \(\sum_{r=1}^{s} \mu_r y_{rO} = 1\) and \(- \sum_{i=1}^{m} v_i x_i + \sum_{r=1}^{s} \mu_r y_{rO} + t_j = 0, j \in E\) correspond to (2), \(\sum_{i=1}^{m} v_i x_{iO} = 1\) reflects the fact that the DMU being evaluated is efficient and \(M\) is a big positive value.

Interpreting the solution in geometric terms we say that in solving (3) we will be selecting optimal weights from the dual (multiplier model) formulation only those which are associated with hyperplanes that are supported by the maximum number of extreme efficient DMUs. The rest of the DMUs usually have unique optimal solutions for their weights.
DMUs. Therefore, the weights so obtained will be the ones corresponding to the facets of the efficient frontier of highest dimension that the unit under assessment contributes to span. In this sense, these will have the maximum possible support from the data from among the alternate optima associated with the current portion of the efficient frontier. In particular, if the assessed unit is located on a full dimensional “efficient (or effective) facet” (FDEF) then the weights will be selected for their association with such a facet of the frontier.

In the second step, we solve the following MILP problem

\[
\begin{align*}
\text{max} & \quad z_o \\
\text{subject to} & \quad \sum_{r=1}^{s} \mu_r y_{ro} = 1 \\
& \quad \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} \mu_r y_{ro} + t_j = 0 \quad j \in E \\
& \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \quad t_j - Mb_j \leq 0 \quad j \in E \\
& \quad \sum_{j \in E} b_j = b_o^* \\
& \quad v_i x_{io} \geq z_o \quad i = 1, \ldots, m \\
& \quad \mu_r y_{ro} \geq z_o \quad r = 1, \ldots, s \\
& \quad b_j \in \{0,1\}, t_j \geq 0 \quad j \in E \\
& \quad z_o \geq 0 \\
& \quad v_i \geq 0 \quad i = 1, \ldots, m \\
& \quad \mu_r \geq 0 \quad r = 1, \ldots, s
\end{align*}
\]

(4)

where \(b_o^*\) is the optimal value of the objective in (3).

Thus, from the weights chosen in the first step, this second step MILP problem selects those which maximize the relative value of the variable with minimum value for the corresponding “virtual” input or output that is represented by \(v_i x_{io}\) and \(\mu_r y_{ro}\). In this step, we look for weights that have associated programs of performance in which the inputs and outputs globally maximize their relative “importance” (see Thanassoulis, 2001).

Cooper, Ruiz and Sirvent (2005) show that, in the case of the efficient DMUs, the weights provided by this two-step procedure are strictly positive, and so, this procedure
makes it possible a portrayal of the efficiency of the unit under assessment in which no variable is completely ignored.\(^2\)

We also note that this procedure can be adapted in a straightforward manner to situations in which efficiency is evaluated by means of AR models (as will be the case in this paper) by simply introducing the corresponding restrictions on the weights both in (3) and (4). In addition, in order to use the two-step procedure above for the case of AR models, it is also necessary to replace the set \(E\) of extreme efficient DMUs with the set of extreme efficient DMUs that are AR efficient (see Cooper et al. (2000) for the definition of AR efficiency).

In particular, in our application to the “effectiveness” evaluations, where only outputs are considered, the restrictions on the weights used in order to incorporate into the analysis the views of the basketball experts will be a set of AR-I type constraints of the form

\[
l_{r,r'} \leq \frac{\mu_{r'}}{\mu_r} \leq u_{r,r'}
\]

where \(\mu_{r'}\) and \(\mu_r\) are the weights of outputs \(r'\) and \(r\) respectively and \(l_{r,r'}\) and \(u_{r,r'}\) are the lower and upper bounds on the allowable values of the ratios of these weights. Therefore, models (2), (3) and (4) will additionally contain these constraints in their formulations when used in our application below.

3. An application to the assessment of basketball players

As noted earlier, the Spanish Premier Basketball League (called ACB) utilizes a scalar index of assessment of players that results from aggregating the classical indicators of performance statistics (points, rebounds, assists...). In this aggregation, the variables that represent positive aspects of the game of a player (e.g., points or rebounds) are weighted with +1, whereas those that represent negative aspects (e.g., turnovers) have a weight of −1.

One can immediately see weaknesses in this index. Perhaps the most important one is the fact that all of the factors are considered to have the same “importance” as a consequence of assigning a weight of magnitude 1 to each of them. However, people familiar with

\(^2\) As a result, the \(\varepsilon > 0\) conditions in (1) and (2) are superfluous.
basketball would generally agree that fouls are not as important as either points or rebounds. This approach also does not take account of the position that the player plays and hence fails to reflect the fact that, for example, rebounds are not as important for a playmaker as they are for a center.

By contrast, DEA allows us to set some requirements on the relative value of the indicators (by using weights restrictions) so that players playing in the same position will be assessed with reference to a previously specified profile (e.g., restrictions that incorporate the preferences on the indicators for players that play in the position of center). And, in addition, DEA allows the weights to vary across players (within the previously established limits), which makes it possible that each player exploits his own characteristics in order to obtain a performance index in the best possible light.

Furthermore, DEA provides us with information on sources and amounts of inefficiency or ineffectiveness for each player and thereby points to where improvements in performance can be made. The result is an evaluation of each player with reference to the rest of the assessed players. For efficient players, the optimal weights will provide an insight on how they achieved their effectiveness. One purpose of this paper is to show how a DEA assessment of basketball players might be used in place of the classical basketball indexes now being used and, more importantly, that the proposed new methodology also provides information that is not available from the classical measures.

The data have been taken from http://www.acb.com/ and correspond to the 2003-2004 season. We have selected a sample of 172 players consisting of those who have played at least 17 games (half a regular season). The idea is to consider only players who have played a large enough number of games to reliably reflect their performances. These 172 players have been classified into the five following groups according to their position: playmaker, guard, small forward, power forward and center. The idea is to have homogenous samples when assessing the efficiency of the players. With respect to the selection of variables, we have defined the following as indicators for the main aspects of the game: shooting, rebounding, ball handling and defense. In particular, the proposed summary of indicators to be included in the model has made possible an important reduction of the dimensionality of the output space compared to the large number of factors used by the ACB index. Other less relevant variables, such as blocked shots, have not been considered. To be specific, the following variables were used for our analyses.
Adjusted Field Goal (AFG) = (PTS – FTM) \times AFG\%, where PTS = points made (per game), FTM = free throws made (per game) and AFG\%, called “adjusted field goal percentage,” is defined as $\frac{PTS − FTM}{2 \times FGA}$, where FGA is the number of field goal attempts.

AFG\% is used in NBA statistics (see http://sports.espn.go.com/nba/statistics/) for the purpose of measuring “shooting efficiency by taking into account the total points a player produces through his field goal attempts. The intuition behind this adjustment is largely to evaluate the impact of “three-point shooting”. Therefore, AFG is a shooting indicator adjusted for opportunities. We could have separately considered PTS-FTM and AFG\% but we preferred to aggregate both variables into AFG in order to avoid mixing a percentage with a volume measure (see Dyson et al. (2001) for a discussion of the pitfalls that can be encountered in DEA applications).

Adjusted Free Throw (AFT) = FTM \times FT\%, where FT\% is the free throw successes percentage. Our comments on the mix of percentages with volume measures is also applicable to this variable.

Rebounds (REB): the number of rebounds per game.

Assists (AST): the number of assists per game.

Steals (STE): the number of steals per game.

Inverse of turnovers (ITURN). We have used the inverse of the number of turnovers per game in order to treat the information regarding this indicator as an output, instead of an input, and thereby also obtain an index with the same form as the one used by the ACB league.

Non-made fouls own (NFO) = 5 – FO, FO being the number of fouls made (per game) by the assessed player. The purpose of this transformation is the same as in the previous variable, ITURN.

Fouls opposite (FOPP): the number of fouls per game the opposite players have made on the player that is being assessed.

In this application, we fortunately had access to the technical staff of Etosa Alicante, the team of the ACB league of the Lucentum Basketball Club of the city of Alicante. This made it possible to have available the opinions of experts for use in developing and
responding to our analysis. To be specific, the members of the technical staff of Etosa Alicante provided us with information regarding the relative value of the above defined variables for each of the different positions. In particular, the coach of Etosa was asked to give his opinion on the relative “importance” of the selected variables for the different positions. For each position, this resulted in a set of AR-I type restrictions.

Thus, the effectiveness of players was assessed by using an output-oriented AR-I model with constant returns to scale, with the selected variables as outputs while including only a nominal input constraint. Thus, the used AR models in this context provide an analysis of the effectiveness of players within limits obtained from a previously specified profile. Finally, since the effectiveness score we use for each player \( \phi^* \) is greater than or equal to unity, the assessment index is defined by \( I^*_0 = \phi^{*-1} \).

The results of the application will be shown below. As representative cases, only the performances of playmakers and centers are reported in order to simplify the discussion and keep the length of this paper within reasonable bounds.

### 3.1. Assessment of the playmakers

The sample of playmakers consists of 41 players. In the opinion of the Etosa coach about the relative value of the factors involved when assessing a playmaker, the following are the most important ones: (a) field goal shooting and (b) the free throw points (and consequently the fouls made by the opposing players on them) as well as (c) the assists and (d) the steals as aspects concerning ball handling and defense. He also stated that the fouls made by a playmaker are a factor of little importance. This led us to include the following set of constraints in (2), so that this became an AR-I-type formulation.

\[
\begin{align*}
\mu_{ITURN} & \leq \mu_{AFG} \\
\mu_{ITURN} & \leq \mu_{AFT} \\
\mu_{ITURN} & \leq \mu_{AST} \\
\mu_{ITURN} & \leq \mu_{STE} \\
\mu_{ITURN} & \leq \mu_{FOPP}
\end{align*}
\]
\[
\begin{align*}
\mu_{REB} & \leq \mu_{AFG} \\
\mu_{REB} & \leq \mu_{AFT} \\
\mu_{REB} & \leq \mu_{AST} \\
\mu_{REB} & \leq \mu_{STE} \\
\mu_{REB} & \leq \mu_{FOPP}
\end{align*}
\]
\[
\begin{align*}
\mu_{NFO} & \leq \mu_{REB} \\
\mu_{NFO} & \leq \mu_{ITURN}
\end{align*}
\]  

(6)

To simplify the notation, we maintain \( \phi^* \) as the optimal value of the AR models used. Obviously, this will not be the optimal value of (1), but rather the one corresponding to the dual problem, (2), once the AR-I constraints have been incorporated into this latter problem.
Table 1 presents evaluations of player performances. In the first column we record 
\( I_0^* = \phi_0^{* -1} \) as a measure of the degree of effectiveness, while in the remaining eight columns we portray profiles for the performance deficiencies that are obtained as explained in the following. It is to be noted that when using AR models the coordinates \( \hat{y}_{r0}, r = 1, ..., s \) of an efficient (=effective) projection point used to evaluate DMU<sub>0</sub> are given by the corresponding envelopment formulation as 
\( \hat{y}_{r0} = \sum_{j=1}^{n} \lambda_{j}^{*} y_{j} \), where \( \lambda_{j}^{*} \) are the values of the intensities at an optimum. Therefore,

\[
\delta_{r0}^* = \hat{y}_{r0} - y_{r0} = \sum_{j=1}^{n} \lambda_{j}^{*} y_{j} - y_{r0}, \quad r = 1, ..., s
\]

provides an estimate of the amount of inefficiency (=lack of effectiveness) in each output for DMU<sub>0</sub>. To better interpret these quantities we transform them into percentages by computing 

\[
\frac{\delta_{r0}^*}{y_{r0}} \times 100\%
\]

These values for each output are the quantities recorded under the corresponding column, and these columns are shown ordered from left to right according to the relative importance of the variables and the heavy lines bordering them group them in this same manner.

Turning to column one, under “Index,” we note that the first four players listed as fully effective, with values of \( I_0^* = 1 \), consists of Bennett, Bullock, Prigioni and Sanchez. As required for full effectiveness, these players also have all slacks equal to the zero values shown in the remaining columns. We therefore lay these players aside for analysis in subsequent tables and now deal with the lack of effectiveness in the different attributes considered for the other players as recorded in Table 1.

As can be seen, these values supply a lot more information than is available only from the \( I_0^* \) scores recorded for effectiveness in column one. (This was also noted by the Etosa coach). For instance, we can see that the main weakness for Turner is that he scores relatively few of the points he should have made from the free throw line, as shown by the value of

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4 See, for instance, Cooper et al. (2000, p. 152-155) for details on definitions and calculations in AR models.
44.01% which is the measure of this shortfall shown under AFT (adjusted free throws). This is partially due, however, to the fact that opposing players make very few personal fouls on this player as indicated by 70.82% under FOPP (fouls by opposing players).

We similarly find that Oliver and Rodríguez should considerably improve their field goal scoring, as is indicated by the values of 117.79% and 139.15% listed for these attributes under AFG. Comas, on the other hand, should mainly increase his AST (the number of assists per game) score by 86.25% while Montecchia should make an effort to improve both AFT (free throw scoring) by 242.63% and FOPP (fouls per game by opposing players) by 246.51% as well as his ball handling since his shortfall in this aspect of his game (AST) stands at 114.85%. Montañez also falls short as a playmaker in this latter variable (AST) by 148.43%.

Table 1

This information was found to be interesting and useful by the ETOSA coach, as we previously remarked. However, no such detail is available from the dual and the envelopment formulations of the AR model used for the fully effective players. We therefore turn to models (3) and (4) to obtain information on these components of their games that are recorded in Table 2.

Table 2 shows the virtual values and records the weights (or multipliers) assigned to these output components, as obtained from (3) and (4) --once (6) is added to the corresponding sets of constraints immediately below them. This is done in the two rows labeled (4) --for model (4), above-- for each player. A comparison with the corresponding values obtained from the EMS DEA computer code are then shown in the next two rows for each player. As can be seen the former --i.e., the row labeled (4)-- shows all of these values to be positive whereas the latter --i.e., the EMS code-- is replete with zeros so that the results from this code fail to provide a full profile in every case. In fact, in three of the four cases, the reported value simply confirms the already determined effectiveness of the performances with a value of $w^*_r y_{ro} = 1$ for only one output and all other $w^*_r = 0$.

It should be noted that the virtual output values in each row sum to 1 for each player and represent the components of their effectiveness score. Thus, we conclude from the results in the first row that Bennet is a very complete player in the sense that he is good in all aspects
of the game, since he exhibits high values in the most important factors. In particular, Bennet is a good scoring player, as is shown by his score of 0.50933 under AFG. For comparison we note that the weights provided by the EMS software give a different picture of Bennet’s game by putting zero weights on AFT and STE and thereby indicate that this player neglects such aspects of the game --which is in contrast to expert opinion. Unlike Bennet, Sánchez and Prigioni can be classified as specialists. To be specific, Sánchez achieves his effectiveness because of his having a high value of 0.92836 in AST (assists), while Prigioni achieves effectiveness because of his 0.93021 value under STE (steals). These results are consistent with what is well known in Spanish basketball: Sánchez is a very good playmaker director whereas Prigioni is probably the best defensive player. Finally, the weights for Bullock provided by the Cooper, Ruiz and Sirvent procedure confirm--as is again well-known--that this playmaker is mainly a good scoring player but does not neglect the other aspects of his game. Again, this is in contrast to the EMS results, where weight is put solely on AFG (adjusted field goals). Results like these underscore the need for selecting weights from among the alternate optimal solutions, which can bring out all aspects of a player’s performance --as is exhibited here by the contrast between results from the EMS software and the Cooper, Ruiz and Sirvent procedures.

Table 2

In order to avoid possible effects from scaling of the variables, such as can occur when using AR-constraints, the above analysis was repeated by replacing the AR-I constraints with the “contingent weight restrictions” suggested in Pedraja-Chaparro et al. (1997). For this purpose each of the coupled variables in (6) --say, for instance, REB and AFG-- the constraint \( \mu_{\text{REB}} \leq \mu_{\text{AFG}} \) must be replaced with the following group of constraints,

\[
\mu_{\text{REB}} y_{\text{REB},j} \leq \mu_{\text{AFG}} y_{\text{AFG},j}, \quad j = 1, \ldots, 41.
\]  

which is equivalent to the constraint

\[
\frac{\mu_{\text{REB}}}{\mu_{\text{AFG}}} \leq \min \left\{ \frac{y_{\text{AFG},j}}{y_{\text{REB},j}}, \quad j = 1, \ldots, 41 \right\}.
\]
When this was done the main conclusions we previously drew remained unaffected. Only very little changes were evidenced in the estimates of both the virtual values and the weights and in only a very few players.

The just described results can be viewed as resulting from an analysis in which the basketball players are assessed with reference to a pre-specified profile that is reflected in the weight restrictions that, as previously noted, set the preferences on the game of a player playing in a given position. This might also be refined or altered for other purposes. For instance, a basketball team might be interested in a player with a specialized profile for a given position. In our experience with Etosa Alicante, it was suggested, among other things, that there was a need for assessing the playmakers according to a scoring profile. The basketball experts found this ability of the DEA methodology especially attractive for the assessment of players when the managers are interested in hiring players as members of a team. To investigate the sample of playmakers with respect to a scoring profile that accords with the opinion of the Etosa technical staff we therefore replaced (6) with the following set of constraints in the DEA model (which again leads to an AR-I-type formulation),

\[
egin{align*}
\mu_{ITURN} & \leq \mu_{AFG} & \mu_{STE} & \leq \mu_{AFG} & \mu_{AST} & \leq \mu_{AFG} & \mu_{REB} & \leq \mu_{AST} \\
\mu_{ITURN} & \leq \mu_{AFT} & \mu_{STE} & \leq \mu_{AFT} & \mu_{AST} & \leq \mu_{AFT} & \mu_{REB} & \leq \mu_{STE} & \mu_{NFO} & \leq \mu_{REB} \\
\mu_{ITURN} & \leq \mu_{FOPP} & \mu_{STE} & \leq \mu_{FOPP} & \mu_{AST} & \leq \mu_{FOPP} & \mu_{REB} & \leq \mu_{ITURN}
\end{align*}
\] (10)

Table 3 records the new indexes \( I_o^* \) and the associated directions of improvement corresponding to the assessment of playmakers with the scoring profile in (10) (with the variables again grouped between the solid lines corresponding to a decreasing order of relative importance). As column one (under “Index”) shows, Sánchez and Prigioni are no longer effective and Prigioni, in particular, is now very deficient. This was to be expected since, as concluded before, these players are specialists in ball-handling and defense, respectively, as witness their very high virtual output values under STE (steals) and AST (assists) in Table 2. They therefore become deficient in effectiveness, as represented by their \( I_o^* \) value, when the most important aspects of the game are directed to the attributes of scoring. In fact, we see that they need to substantially improve the attributes represented by AFG, AFT and FOPP in the first three columns, within the heavy lines, that are now regarded
as most important. This can also be concluded for some of the playmakers previously rated as ineffective. See, for instance, Montecchia, whose $I^*_o$ score has dropped from 0.81 in Table 1 to 0.67 in Table 3 and this reemphasizes his need to improve his scoring (in particular from the free throw line). Nevertheless, regarding this new scoring profile, he shows better performances than before in other aspects of his game that are more concerned with teamwork, such as assists, steals or turnovers. Note also the negative value of the deficiencies recorded for these variables in the last two columns, which indicate that he exceeds the values considered as effective for this player in these aspects of the game.

Table 3

3.2. Assessment of centers

Evidently these profiles depend on the types of position as well as the attributes to be employed, such as “playmakers.” We now show this for the position of centers. The sample of centers consists of 44 players. To assess their effectiveness, we need to modify the set of weights restrictions we used in the case of playmakers. Again, this was done in a manner that remains within the bounds of the opinions of members of the Etosa technical staff. For centers, they believed that rebounds are one of the most important player qualities. Moreover, in the opinion of the Etosa coach, the aspects concerning ball handling, such as the assists and the turnovers, are not particularly important for a center. These types of considerations led us to incorporate the following set of restrictions on the weights in (2) in order to specify the center profile

\[
\begin{align*}
\mu_{\text{STE}} & \leq \mu_{\text{AFG}} \\
\mu_{\text{STE}} & \leq \mu_{\text{AFT}} \quad \mu_{\text{AST}} & \leq \mu_{\text{STE}} \quad \mu_{\text{NFO}} & \leq \mu_{\text{AST}} \\
\mu_{\text{STE}} & \leq \mu_{\text{REB}} \quad \mu_{\text{ITURN}} & \leq \mu_{\text{STE}} \quad \mu_{\text{NFO}} & \leq \mu_{\text{ITURN}} \\
\mu_{\text{STE}} & \leq \mu_{\text{FOPP}}
\end{align*}
\]  

The results of the new AR formulations are shown in Table 4. As a consequence of the new set of constraints (11), we have moved REB (rebounds) from the third to the first set of bold lines in this table. Besides, we maintain only STE (steals) in the second set of bold lines and transfer AST and ITURN to a third level of importance in the next to last set of bold
lines. From this table we can see that five players were found to be fully effective: David, Garcés, Kambala, Scott, and Thompson. As with playmakers, Table 4 records the index $I^*_o$ of the top ten assessed players and their corresponding possibilities for improvement in the case of the not fully effective players. We can see that Reyes as a center has his main weakness in the free throw line as shown by a value of 30.90% under AFT, while he outperforms the estimated levels for effectiveness in other aspects of the game considered less important for a center (see the values -35.29%, -12.34% and -16.73% under STE, ITURN and NFO, respectively). This same difficulty regarding the free throw line is evidenced even more in the case of Tabak who does not seem to provoke fouls from opposing players. The same is true for Bramlett, and even more so in the case of Tomasevic. In confirmation we may say that it is well-known that the latter player has very low free throw percentages. Besides, this AR analysis detects another well-known fact of Spanish basketball expert opinion because, in spite of being a center, Tomasevic is a good player in other aspects such as ball handling and defense, especially in the number of assists -- note the negative percentage of improvement under the variable AST, which means that he exceeds the ability of his referent effective player in this aspect of the game.

Table 4

For the five players with scores of $I^*_o = 1$ in Table 4, we also provide the virtual outputs and the corresponding weights (or multipliers), that are recorded in Table 5. The virtual outputs associated with the weights provided by the Cooper, Ruiz and Sirvent (2005) procedure show that Kambala, Scott and Thompson are centers with complete games, being particularly good scoring players. In addition to high values, in excess of 60% for AFT, they all have relatively good values in all of the most important attributes for centers. Garcés, however, is a specialist in catching rebounds with a virtual value of 0.95403 under REB and, finally, David is at his best from the free throw line as shown by the value of 0.90004 under AFT. Again, the weights provided by the Cooper, Ruiz and Sirvent procedure are seen to provide a more comprehensive portrayal as compared to the optimal weights provided by EMS which, as in the other tables, exhibit numerous zeros.\(^5\) Thus, on the basis of the EMS weights, Kambala would simply be a good scoring center as represented by his EMS scores.

\(^5\) This also happens with other softwares. See the comparisons provided in Cooper, Ruiz and Sirvent (2005).
under AFG and AFT, Scott would be a specialist in free throw points as shown under AFT and FOPP and, finally, Thompson would be very good in rebounds and would not be a bad player from the free throw line. As compared to the scores in the rows labeled (4) in Table 5, however, and, as was the case in Table 1, the procedure used by the EMS computer code provided only partial views of what is well-known, qualitatively, about the games of these players.

Table 5

Finally, we would like to stress that all the results we have obtained in this section are qualitatively consistent with the conclusions we might draw from the ACB index of player assessment. However, it is to be noted that such a comparison by the latter index, if meaningful, could only be made in terms of rankings, since the ACB only provides a scalar index for each player that is calculated by simply aggregating his recorded values in the indicators used in the statistics. Therefore, it is obvious that DEA, in the way we have used it in this application, provides much more information, since the ACB index does not take into account (1) the position where the assessed player plays, (2) which aspects of the game of a player in that position must be analyzed more carefully or (3) the behavior of the rest of players playing the same position in order to determine attainable values in the used indicators that can be used as targets for the players being evaluated. Finally, the use of only (0,1) in the ACB indicator omits any information on the degree of attainment for the attributes of interest.

4. Conclusions

In this paper we have presented an application of DEA to the assessment of basketball players in the Spanish Basketball League. The purpose of this application is twofold: On one hand, we want to show that DEA can be advantageously used as an alternative to the ACB index of player assessment. Even when the results are similar, DEA provides much more information. In addition, and in contrast to the ACB index, we have made a choice of weights by exploiting the flexibility of DEA in the sense that we provide player-specific weights that profile player characteristics and also take into consideration the prior knowledge of the
experts about the relative importance of the indicators used. It is to be noted that the basketball experts appreciated the usefulness of this methodology for the assessment of players. In particular, they especially liked the fact already mentioned that they are able to assess players according to a previously specified profile and then have the procedure go on to provide much more detailed (quantitatively expressed) information.

Of particular interest was the fact that our approach added to the customary uses of DEA by exhibiting whole profiles that underlay the effectiveness score. This makes it possible, for instance, to distinguish between a good all around player from another player with the same effectiveness score but attained because of a high virtual value for one attribute accompanied by low values on the other attributes. This not only shows weaknesses meriting attention for particular players, it also provides information on team qualities that need to be strengthened. Still further such information can be obtained by decomposing the virtual output values into their weight and quantity components as in, for instance, a substitution analysis.

In conclusion we would like to point out that the methodological aspects of DEA we have described here can be extended to many other contexts, including cases in which the focus is not only on defining an index of performance but also on assessing components underlying the $I_o^*$ values.

Acknowledgements

We would like to thank the Lucentum Basketball Club for the opportunities of access to the expert opinions that they provided. We are also grateful to the Ministerio de Educación y Ciencia (MTM2004-07473) for financial support. Support from the IC2 Institute of the University of Texas at Austin is also gratefully acknowledged.
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Table 1. Effectiveness scores and improvement percentages (playmakers)
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<td>Bramlett, A.J.</td>
<td>0.8349</td>
<td>19.78%</td>
<td>183.17%</td>
<td>19.78%</td>
<td>51.56%</td>
<td>4.94%</td>
<td>-41.33%</td>
<td>-1.57%</td>
<td>50.19%</td>
</tr>
<tr>
<td>Reynolds-Dean</td>
<td>0.8309</td>
<td>34.37%</td>
<td>20.36%</td>
<td>31.83%</td>
<td>43.18%</td>
<td>-32.90%</td>
<td>-6.67%</td>
<td>-5.82%</td>
<td>-23.39%</td>
</tr>
<tr>
<td>Tomasevic, D.</td>
<td>0.7935</td>
<td>81.48%</td>
<td>221.10%</td>
<td>26.02%</td>
<td>53.46%</td>
<td>-42.70%</td>
<td>-60.08%</td>
<td>-17.30%</td>
<td>-16.24%</td>
</tr>
</tbody>
</table>

Table 4. Effectiveness scores and improvement percentages (centers)
<table>
<thead>
<tr>
<th>Player</th>
<th>AFG</th>
<th>AFT</th>
<th>REB</th>
<th>FOPP</th>
<th>STE</th>
<th>AST</th>
<th>ITURN</th>
<th>NFO</th>
</tr>
</thead>
<tbody>
<tr>
<td>David, Kornel</td>
<td>0.05444</td>
<td>0.90004</td>
<td>0.02156</td>
<td>0.01202</td>
<td>0.00409</td>
<td>0.00266</td>
<td>0.00260</td>
<td>0.00260</td>
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<tr>
<td></td>
<td>(0.00751)</td>
<td>(0.34480)</td>
<td>(0.00421)</td>
<td>(0.00421)</td>
<td>(0.00421)</td>
<td>(0.00174)</td>
<td>(0.00421)</td>
<td>(0.00174)</td>
</tr>
<tr>
<td>Garcés, Rubén</td>
<td>0.02132</td>
<td>0.00389</td>
<td>0.95403</td>
<td>0.01153</td>
<td>0.00367</td>
<td>0.00185</td>
<td>0.00185</td>
<td>0.00185</td>
</tr>
<tr>
<td></td>
<td>(0.00405)</td>
<td>(0.00405)</td>
<td>(0.09723)</td>
<td>(0.00405)</td>
<td>(0.00405)</td>
<td>(0.00349)</td>
<td>(0.00405)</td>
<td>(0.00108)</td>
</tr>
<tr>
<td>Kambala, K.</td>
<td>0.22280</td>
<td>0.68480</td>
<td>0.04510</td>
<td>0.03441</td>
<td>0.00488</td>
<td>0.00233</td>
<td>0.00335</td>
<td>0.00233</td>
</tr>
<tr>
<td></td>
<td>(0.02413)</td>
<td>(0.26894)</td>
<td>(0.00744)</td>
<td>(0.00744)</td>
<td>(0.00744)</td>
<td>(0.00744)</td>
<td>(0.00744)</td>
<td>(0.00162)</td>
</tr>
<tr>
<td>Scott, Brent</td>
<td>0.24283</td>
<td>0.60236</td>
<td>0.08595</td>
<td>0.05306</td>
<td>0.00663</td>
<td>0.00361</td>
<td>0.00278</td>
<td>0.00278</td>
</tr>
<tr>
<td></td>
<td>(0.02754)</td>
<td>(0.24803)</td>
<td>(0.00940)</td>
<td>(0.00940)</td>
<td>(0.00940)</td>
<td>(0.00189)</td>
<td>(0.00940)</td>
<td>(0.00189)</td>
</tr>
<tr>
<td>Thompson, Kevin</td>
<td>0.21513</td>
<td>0.64024</td>
<td>0.08340</td>
<td>0.04093</td>
<td>0.00824</td>
<td>0.00402</td>
<td>0.00402</td>
<td>0.00402</td>
</tr>
<tr>
<td></td>
<td>(0.02655)</td>
<td>(0.25424)</td>
<td>(0.00875)</td>
<td>(0.00875)</td>
<td>(0.00875)</td>
<td>(0.00370)</td>
<td>(0.00875)</td>
<td>(0.00214)</td>
</tr>
</tbody>
</table>

Table 5. Virtual outputs and multiplier weights for centers.